



NVIDIA®

Variance Shadow Maps

Andrew Lauritzen, RapidMind

Overview of Shadow Mapping



- **Introduced by Williams in 1978**
- **Advantages compared to shadow volumes:**
 - **Cost less sensitive to geometric complexity**
 - **Can be queried at arbitrary locations**
 - **Often easier to implement**
- **Disadvantages:**
 - **Aliasing**

Shadow Mapping Algorithm

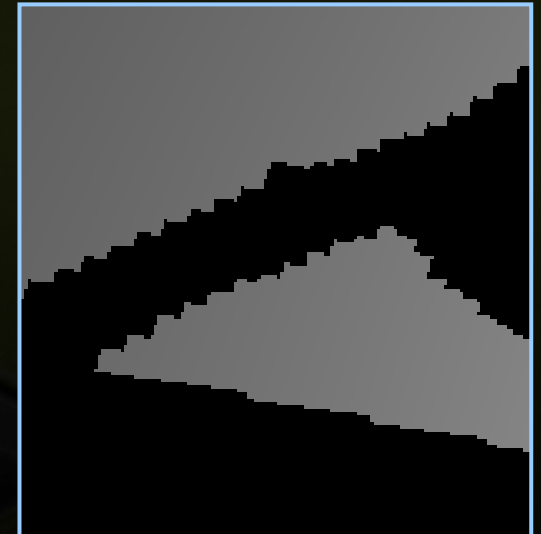
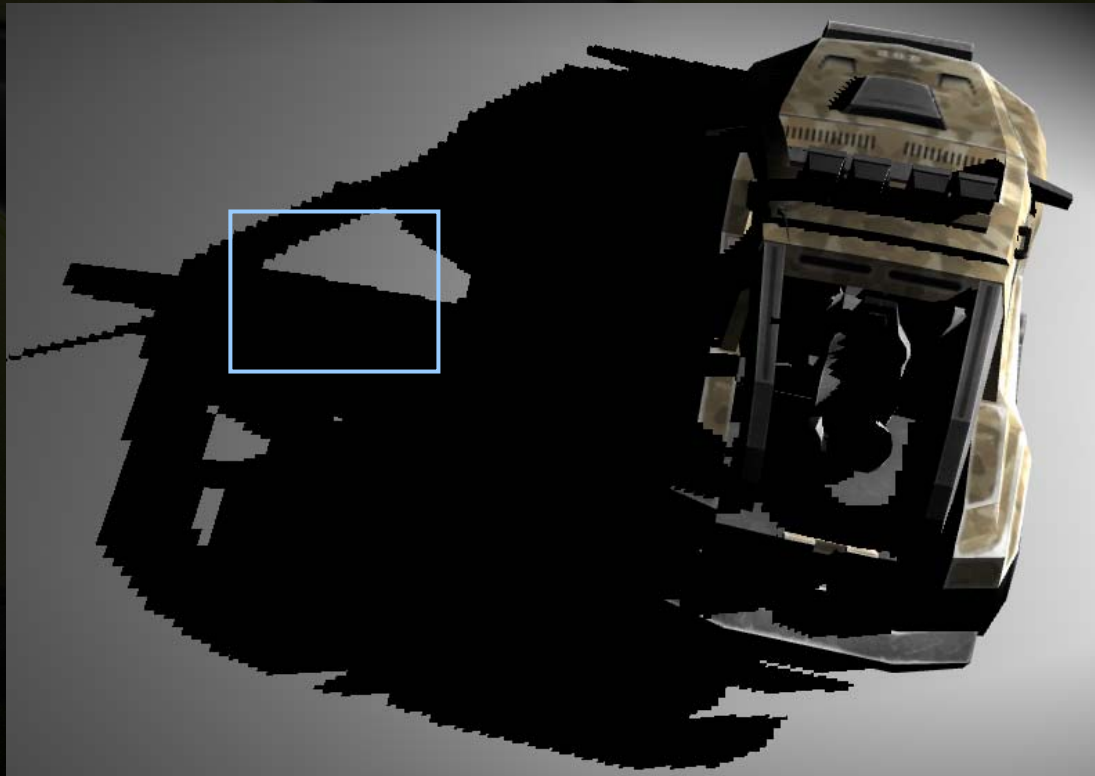


- **Render scene from light's point of view**
 - **Store depth of each pixel**
- **When shading a surface:**
 - **Transform surface point into light coordinates**
 - **Compare current surface depth to stored depth**
 - **If depth $>$ stored depth, the pixel is in shadow; otherwise the pixel is lit**

Aliasing Artifacts



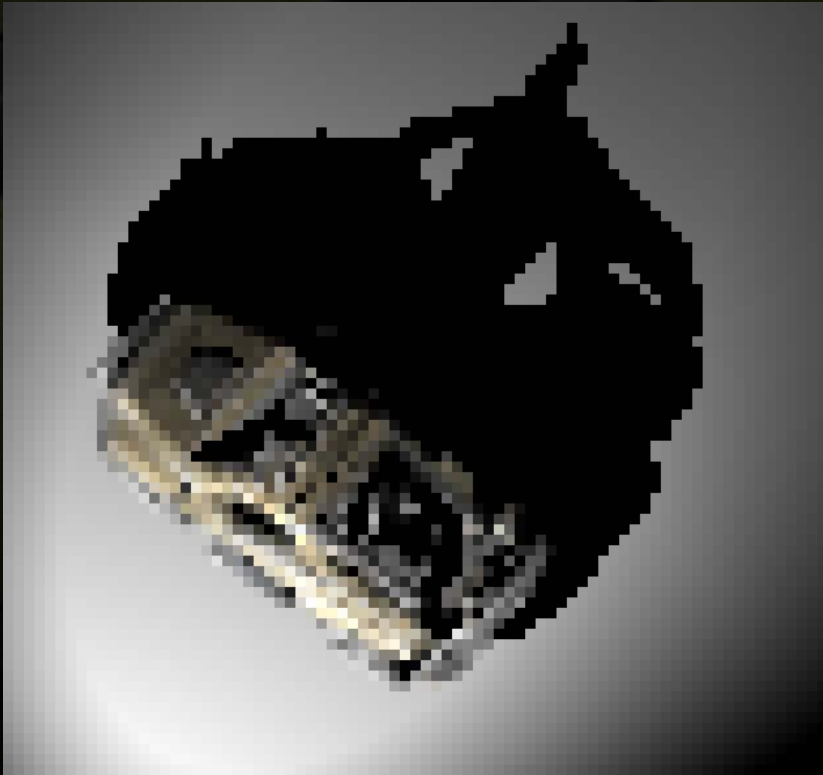
● Magnification artifacts



Aliasing Artifacts



● Minification artifacts



- Typically encountered when viewed from a distance
- Produces ugly and distracting “swimming” effect along shadow edges

Aliasing Artifacts



- **Anisotropic artifacts**
 - A mix of minification and magnification
 - Encountered at shallow angles



Solutions?



- Also encountered with colour textures
- Reduce aliasing by hardware filtering
 - Magnification artifacts => linear interpolation
 - Minification artifacts => trilinear, mipmapping
 - Anisotropic artifacts => anisotropic filtering

Solutions?



- Can we apply these to shadow maps?
 - Not at the moment
- Interpolating depths is incorrect
 - Gives $\text{depth} < \text{average}(\text{occluder_depth})$
 - Want $\text{average}(\text{depth} < \text{occluder_depth})$

Percentage Closer Filtering

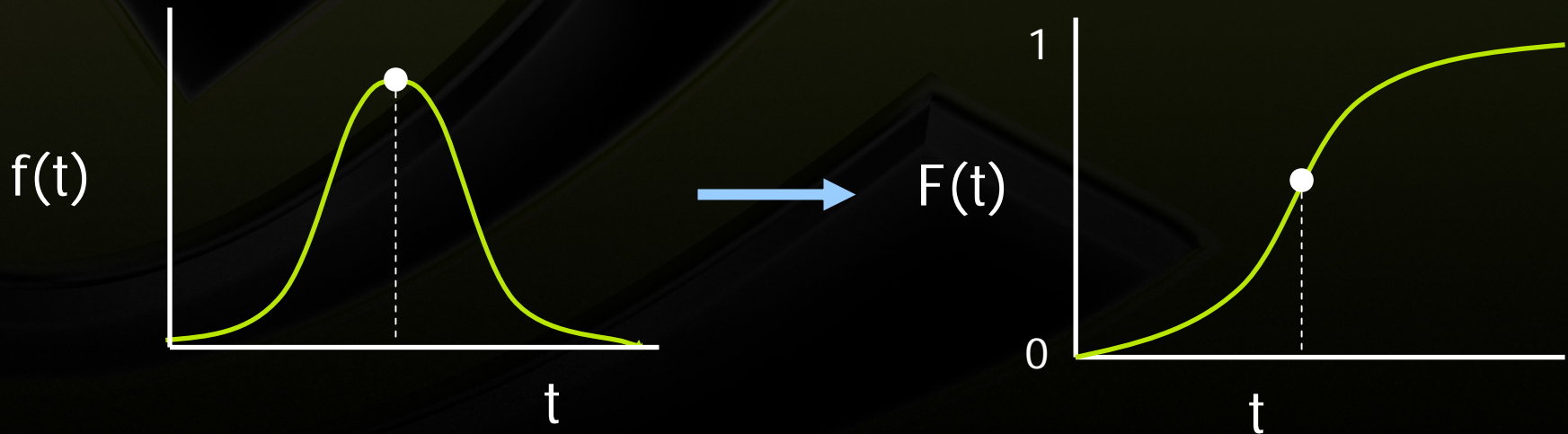


- Proposed by Reeves et al. in 1987
- Filter result of the depth comparison
 - Sample surrounding shadow map pixels
 - Do a depth comparison for each pixel
 - Percentage lit is the percentage of pixels that pass the depth comparison (i.e. are “closer” than the nearest occluder)
- NVIDIA hardware support for bilinear PCF
- Good results, but can be expensive!

Occluder Distribution



- Really want a cumulative distribution function (CDF) of a set of depths
 - $F(t) = P(x \leq t)$
 - $F(t)$ is the probability that a fragment at distance “ t ” from the light is in shadow



Deep Shadow Maps



- Lokovic and Veach, in 2000
- Per-pixel piecewise linear function
- No hardware filtering
- Complex reconstruction

Occluder Distribution



- A representation that filters linearly?
 - Allows us to utilize hardware filtering
- Idea: Moments of distribution function!
 - $E(x)$ is the mean, $E(x^2)$, $E(x^3)$, etc.
 - Linear in distribution

Variance Shadow Maps



- **Store depth squared as well as depth**
 - Gives $E(x)$ and $E(x^2)$ where x is the depth of the nearest occluder
 - Use the moments to approximate the fraction of the distribution that is more distant than the surface point being shaded

Variance Shadow Maps



- We want to find $P(x \geq t)$
- We have the mean, and can find variance:
 - $\mu = E(x)$
 - $\sigma^2 = E(x^2) - E(x)^2$
- Cannot compute CDF exactly
- Chebyshev's Inequality states:

$$P(x \geq t) \leq p_{max}(t) \equiv \frac{\sigma^2}{\sigma^2 + (t - \mu)^2}$$

Variance Shadow Maps



- Inequality only gives an upper bound
 - Becomes equality in the case of single planar occluder and receiver
 - In a small neighbourhood, an occluder and receiver will have constant depth and thus p_{\max} will provide a close approximation to p
- So just use p_{\max} for rendering

Implementation



// Call the parent light shader

```
light_contrib & dir_to_light & dist_to_light & n_dot_l =  
    spot_light_shader(surf_position, surf_normal);
```

// Transform the surface position into light space and project

```
ShAttrib4f surf_light = light_view_projection | surface_position;  
ShTexCoord2f tex_coord = 0.5 * surf_light(0,1)/surf_light(3) + 0.5;
```

// Query the shadow map

```
ShAttrib2f moments = shadow_map(tex_coord);
```

// Standard shadow map comparison

```
ShAttrib1f lit_factor = (dist_to_light <= moments(0));
```

// Variance shadow mapping

```
ShAttrib1f E_x2 = moments(1);  
ShAttrib1f Ex_2 = moments(0) * moments(0);  
ShAttrib1f variance = E_x2 - Ex_2;  
ShAttrib1f m_d = moments(0) - dist_to_light;  
ShAttrib1f p_max = variance / (variance + m_d * m_d);
```

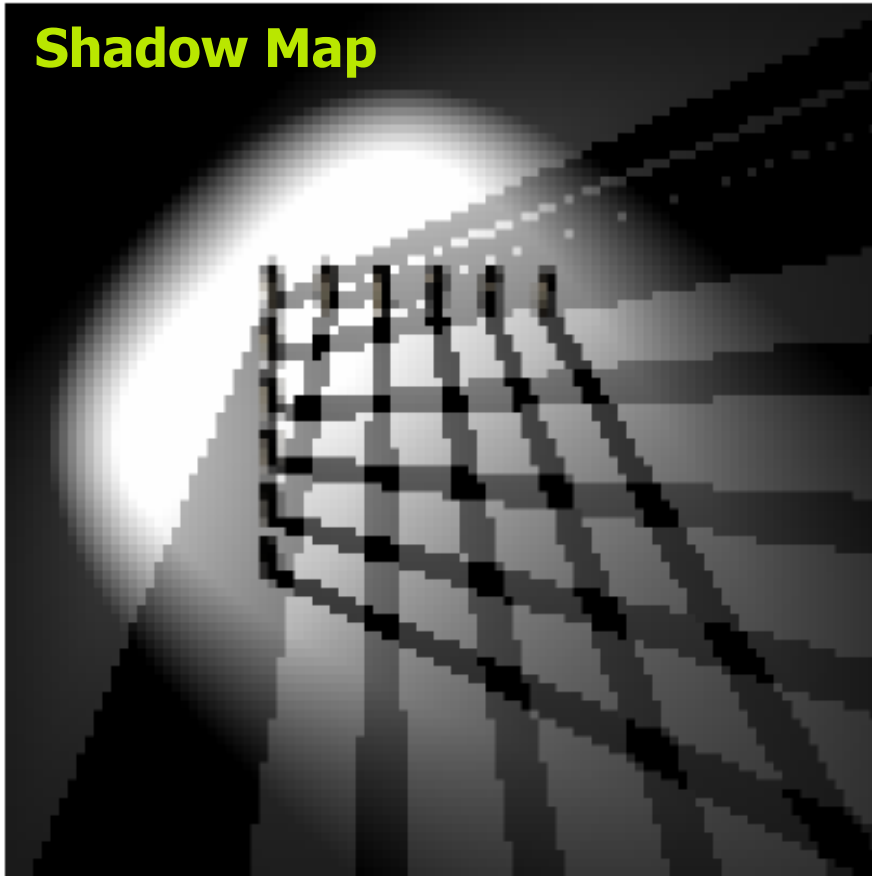
// Attenuate the light contribution as necessary

```
light_contrib *= max(lit_factor, p_max);
```

Mipmapping Results



Shadow Map



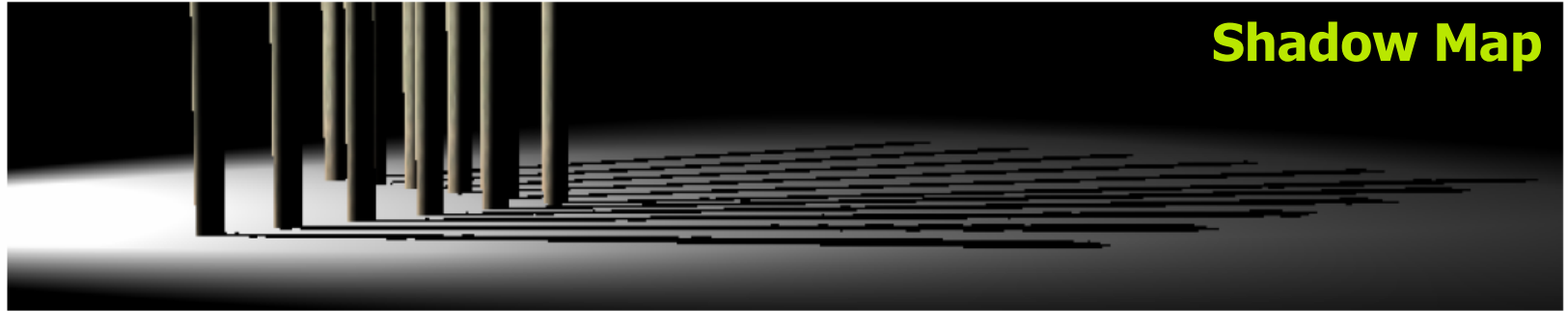
Variance Shadow Map



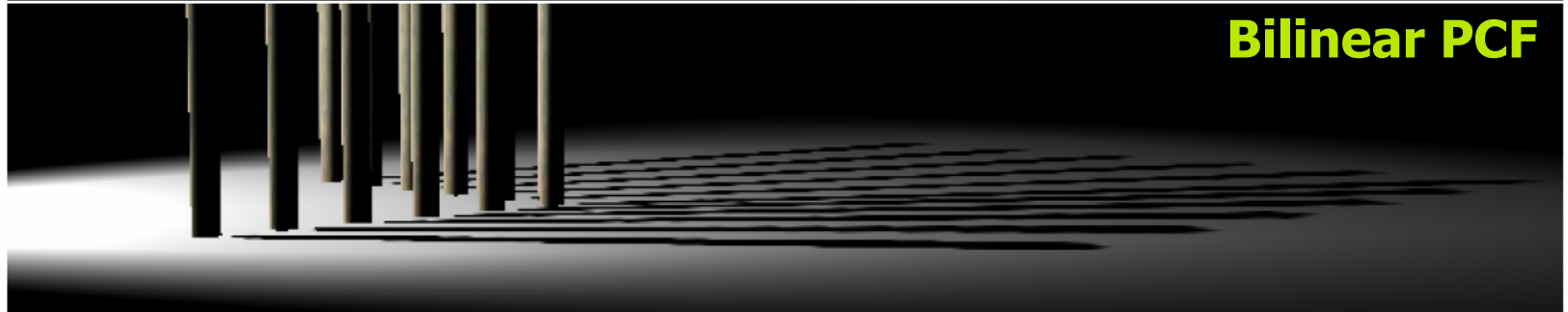
Anisotropic Filtering Results



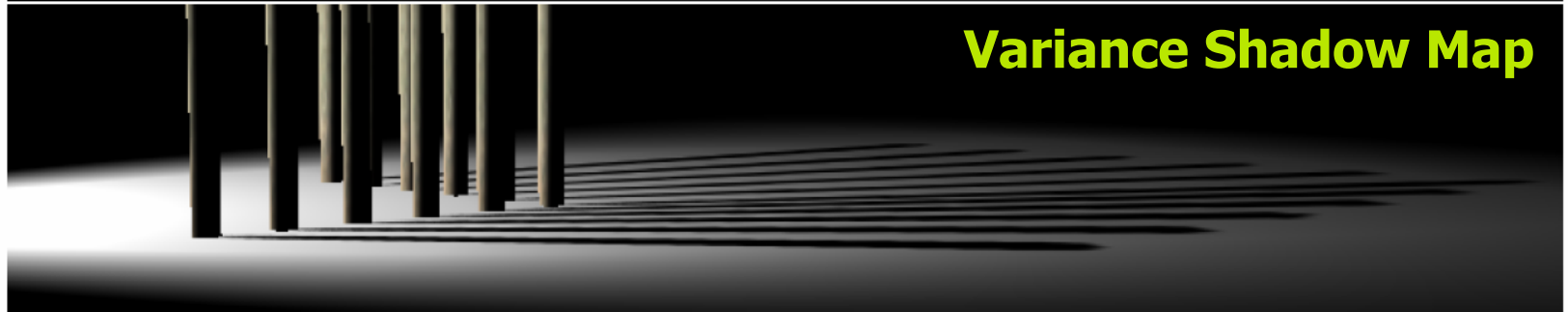
Shadow Map



Bilinear PCF



Variance Shadow Map

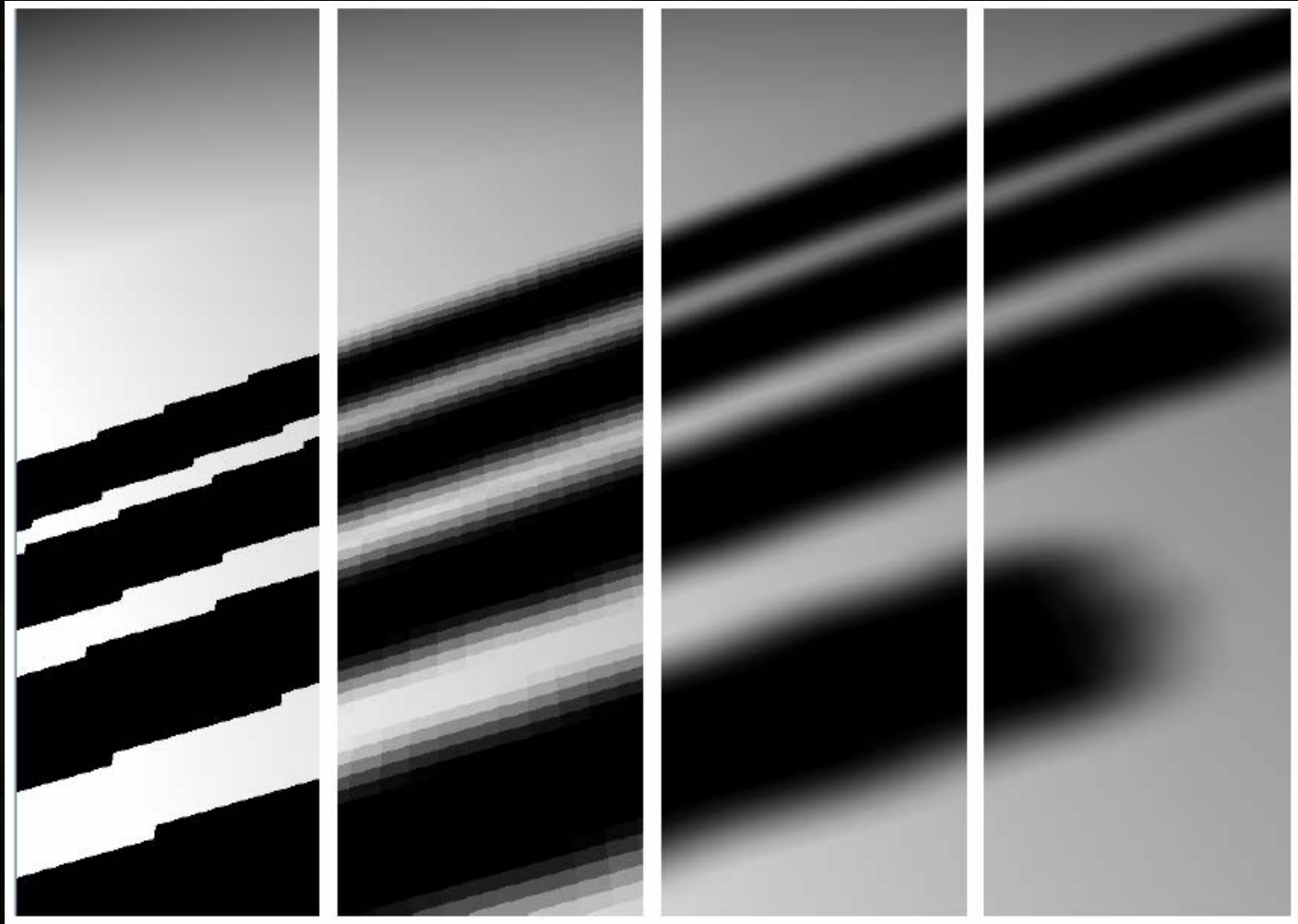


Variance Shadow Maps



- **Can we do more?**
 - **Our shadow maps can be arbitrarily filtered now**
- **Pre-filter shadow map using a Gaussian blur**
 - **Equivalent to percentage closer filtering**
 - **Separable convolution $\Rightarrow O(n)$ on kernel size**
 - **Much faster than PCF complexity of $O(n^2)$**

Gaussian Blur Results



SM

PCF 5x5

Bil.PCF 5x5

VSM

Super-sampling



- **Generate more samples and filter**
 - Render large shadow map and down-sample
 - Or simply use texture LOD bias
- **Tiled rendering of a huge shadow map**
 - Render 4 tiles at 4096x4096 each
 - Down-sample to a single texture
 - Gives an anti-aliased 4096x4096 shadow map

Multi-sampling



- Simply enable multi-sampling while rendering the shadow map
- Support is dependent on chosen texture format
 - More of this later...
- Notes on gamma correction
 - Hardware might “gamma correct” the samples
 - This is incorrect for non-colour data!
 - Ideally we want to turn this “feature” off...

Other Fun Stuff



- **Orthogonal to projection-warping techniques**
 - **Perspective shadow maps (PSM)**
 - **Trapezoidal shadow maps (TSM)**

Demo



Texture Formats



- **Ideal texture format:**
 - **Renderable**
 - **Two components**
 - **High precision**
 - **Supports filtering (anisotropic, mipmapping)**
 - **Supports multisampling**

Depth and Shadow Formats



- **+/- Indirectly renderable**
- **- Single-component**
- **- Often highly non-uniform precision**
- **- Do not support arbitrary linear filtering**
 - Mipmapping, trilinear, anisotropic, etc.
- **- Do not support multisampling**
- **Not the way to go...**

Floating-point Formats



- **No renderable two-component formats!**
- **4x fp16**
 - + NVIDIA GeForce 6/7 supports filtering!
 - +/- Average precision
 - +/- Some hardware supports multisampling
- **4x fp32**
 - + Great precision
 - - No filtering on current hardware
 - - No multisampling on current hardware
- **Probably the best current options**

Fixed-point Formats



- **8-bit formats?**

- - Poor precision makes these unusable

- **2x 16-bit (i.e. G16R16)**

- + Two component
 - + Often supports filtering
 - +/- Renderable on some hardware
 - +/- Acceptable precision (at least as good as fp16)
 - +/- Some hardware supports multisampling

- **Dreaming:**

- 2x 32-bit filterable fixed point format?

Texture Format Summary



- **Floating-point formats probably the best**
 - Ideally we want filterable fp32
- **16-bit fixed-point formats could work too**
 - Dependent on what hardware supports

Numerical Stability



- Recall the computation of variance:

- $\sigma^2 = E(x^2) - E(x)^2$

- Highly numerically unstable!

- Recall Chebyshev's Inequality:

$$P(x \geq t) \leq p_{max}(t) \equiv \frac{\sigma^2}{\sigma^2 + (t - \mu)^2}$$

- Can be a problem when fragment is near occluder

- Need a high-precision texture format

Ways to Improve Stability



- Can use any distance metric that we want
 - Post-projection “depth” (z) is a bad choice
 - Use a linear metric, ex. distance to camera
- When using floating-point formats
 - Rescale the numeric range to fall in $[-1, 1]$
 - Gets an extra bit of precision from the sign

Ways to Improve Stability



- Four-component floating-point formats

- Store extra precision in extra components
- Must still filter linearly!!!

- Example encoding:

```
ShAttrib2f moments = (...);  
ShOutputAttrib4f output;  
const float factor = 64.0f; // Try to gain 6 more bits  
output(0,1) = frac(moments * factor) / factor;  
output(2,3) = moments - output(0,1);
```

- Example decoding:

```
ShAttrib4f input = shadow_map(tex_coord);  
ShAttrib2f moments = input(0,1) + input(2,3);
```


Ways to Improve Stability



- Use a 32-bit per component floating-point texture!
- We've had no precision problems with fp32

Notes on Shadow Bias

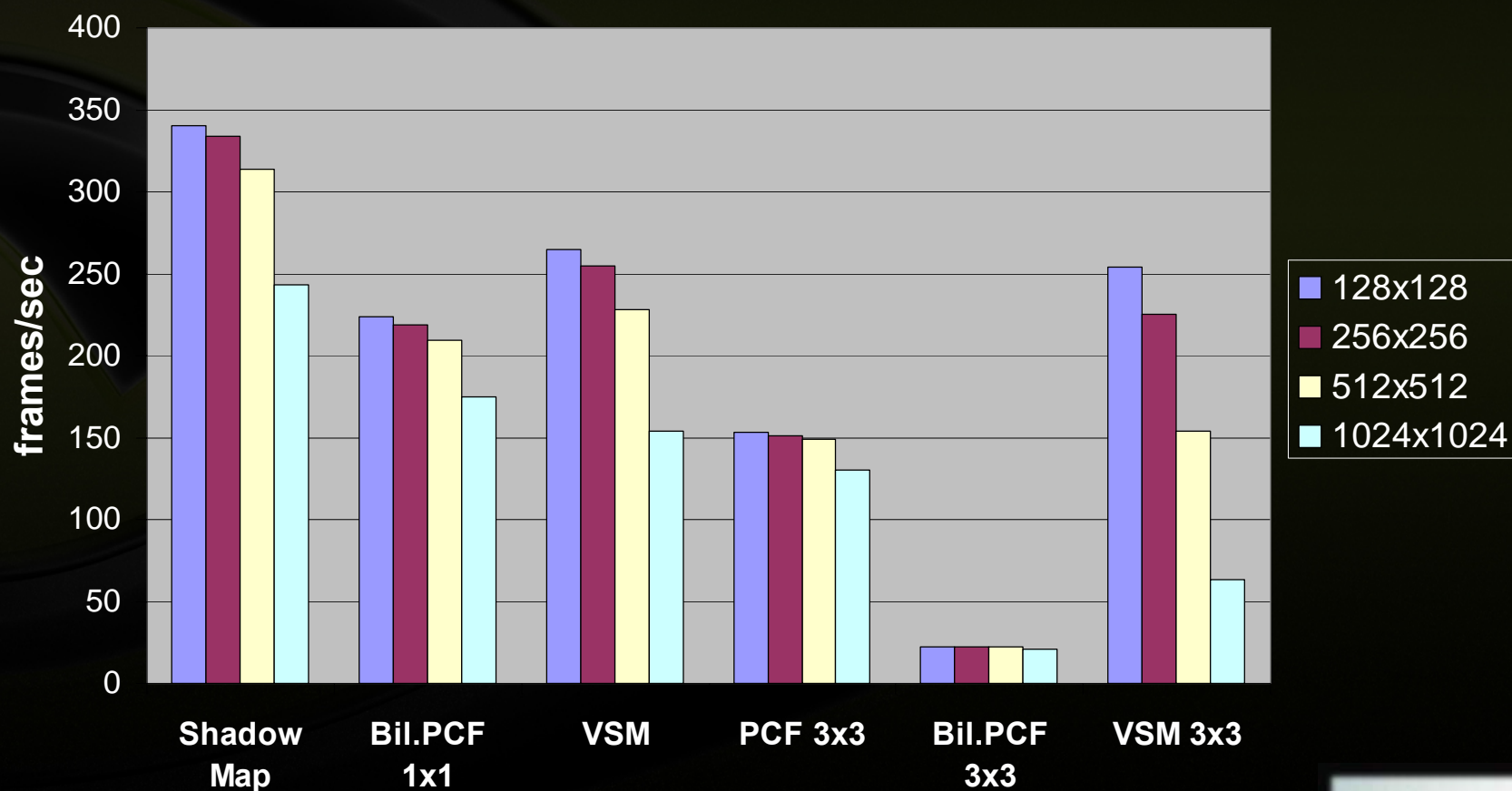


- **Biassing depth comparison usually required**
 - Proportional to slope of polygon (`glPolygonOffset`)
 - Scene dependent and error-prone
- **Not required for variance shadow maps!**
 - If $(t - \mu) \sim 0$ then $p_{\max} \sim 1$
- **May want to bias variance very slightly**
 - For numeric stability reasons
 - This is neither slope nor scene dependent!

How Fast?



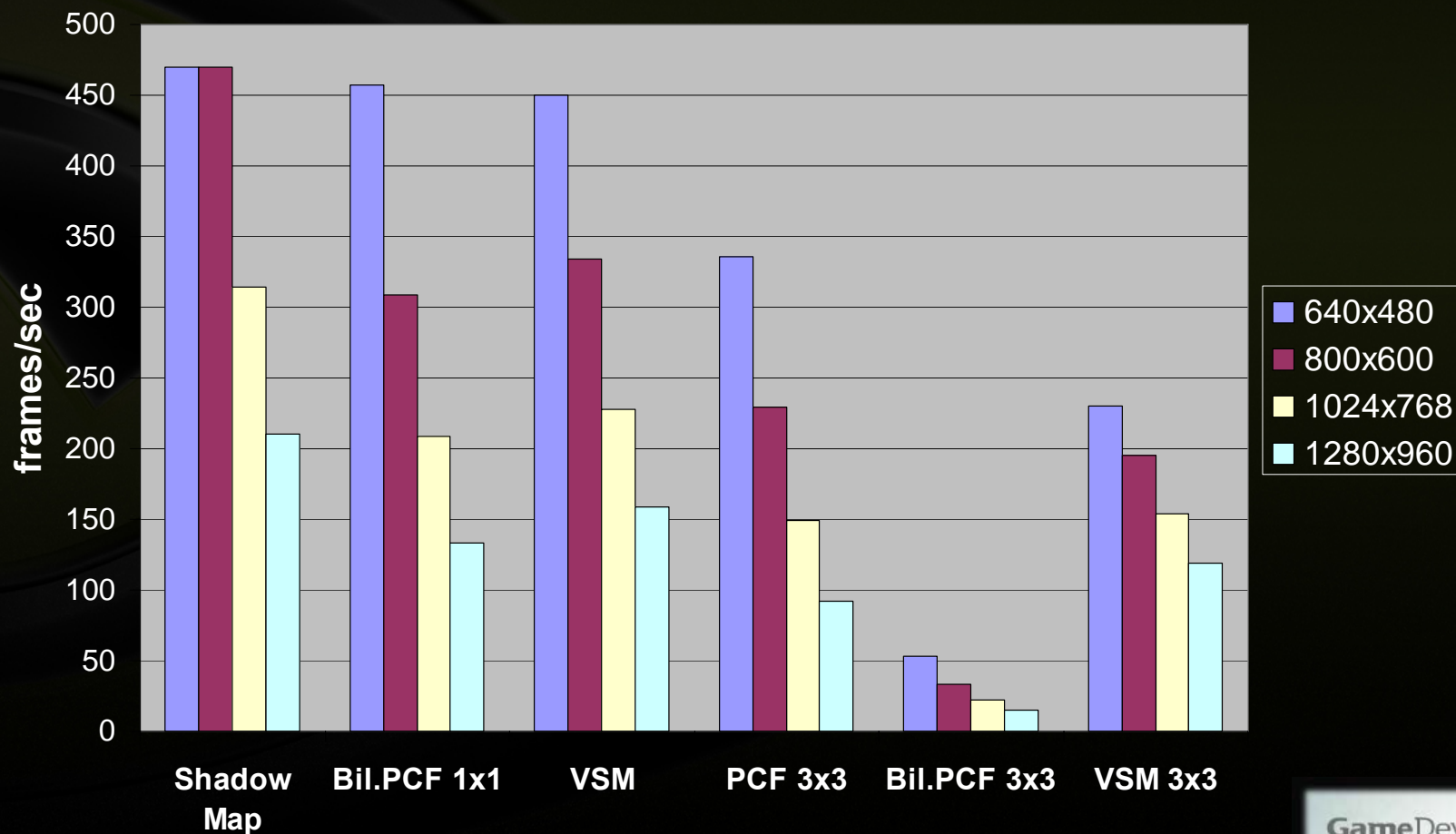
GeForce 6800GT @ 1024x768



How Fast?



● Fix shadow map at 512x512



Light Bleeding



- p_{\max} works in many situations, but not all
- When σ^2 is large, can get “light bleeding” :



Ways to Reduce Light Bleeding



- **Lower depth complexity in light space**
 - Ex. Use variance shadow maps for the sun, not headlights
 - Construct scenes with this artifact in mind
 - Control attenuation ranges carefully
- **Use ambient or multiple lights**
 - Contrast will be lessened
- **Use static lights**
 - Moving lights makes the projection obvious
- **Use smaller filter regions**
 - Artifact is only as large as the filter region

Ultimate Solutions



- Find a higher-order inequality?
- Fast, programmable hardware filtering?
- Combine with percentage closer soft shadows
 - Randima Fernando (NVIDIA), 2005
 - Cheap, perceptually-correct soft shadows?
- Lots of potentially fruitful hybrid techniques!

Conclusion



- **Introduced a simple solution to many forms of shadow map aliasing**
- **Implemented easily on modern hardware**
- **Compares favourably in both performance and quality to existing techniques**

For More Information...



- Donnelly and Lauritzen, *Variance Shadow Maps*, ACM Symposium on Interactive 3D Graphics and Games 2006
- <http://www.punkuser.net/vsm/>
- Questions?

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